

Continuous Random Variables

Lecture 22 Section 7.5.4

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Mon, Feb 27, 2012

Outline

- 1 Random Variables
 - The Uniform Distribution
 - A Non-uniform Distribution
- 2 Hypothesis Testing
 - Triangular Distributions
 - Normal Distributions
- 3 Assignment

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Random Variables

Definition (Random variable)

A **random variable** is a variable whose value is determined by the outcome of a random process.

Definition (Discrete random variable)

A **discrete random variable** is a random variable whose set of possible values is a discrete set.

Definition (Continuous random variable)

A **continuous random variable** is a random variable whose set of possible values is a continuous set.

Continuous Probability Density Functions

Definition (Continuous Probability Density Function)

A **continuous probability distribution function**, or **pdf**, for a random variable X is a continuous function with the property that the area below the graph of the function between any two points a and b equals the probability that $a \leq X \leq b$.

- Remember,

AREA = PROPORTION = PROBABILITY

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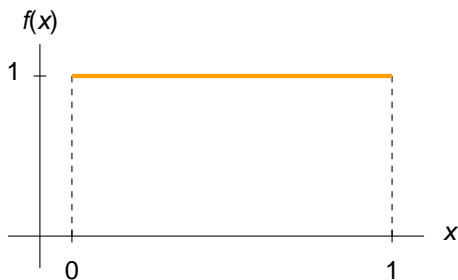
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Example

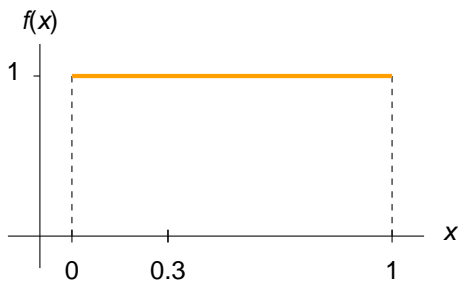
- The TI-83 will return a random number between 0 and 1 if we enter `rand` and press `ENTER`.
- These numbers have a uniform distribution from 0 to 1.
- Let X be the random number whose value is determined by the `rand` function.

Example



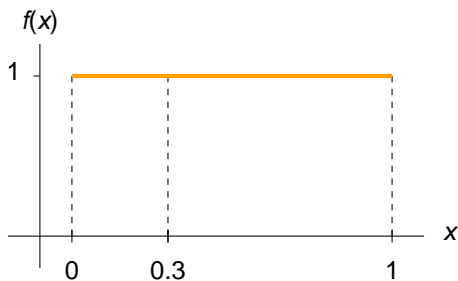
What is the probability that the random number is at least 0.3?

Example



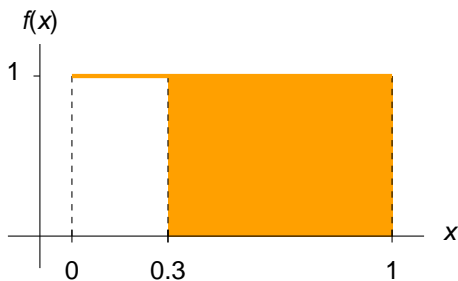
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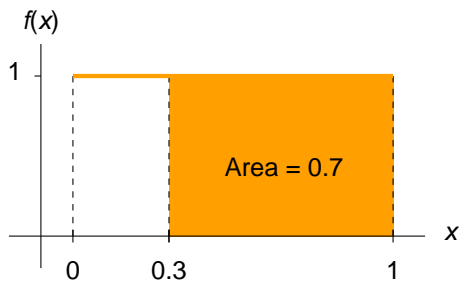
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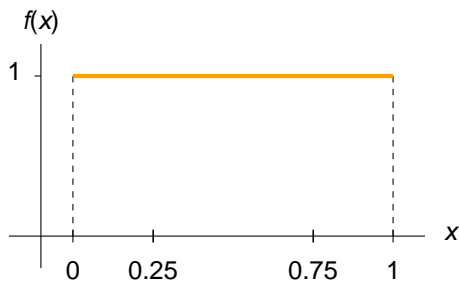
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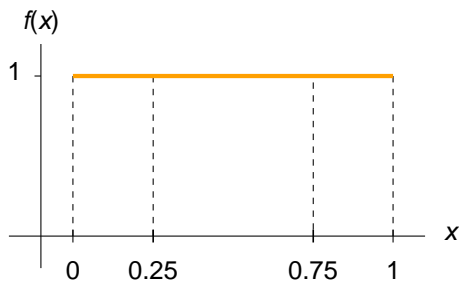
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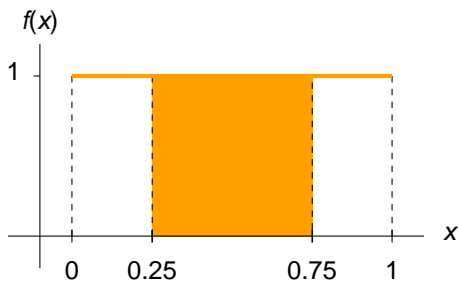
What is the probability that the random number is between 0.25 and 0.75?

Example



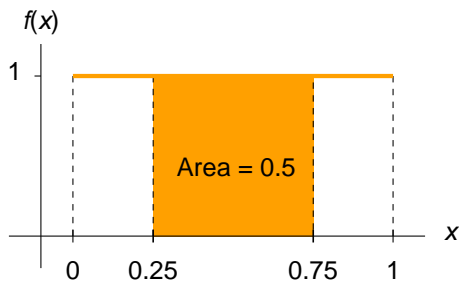
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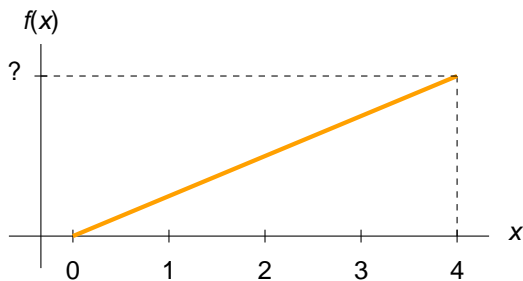
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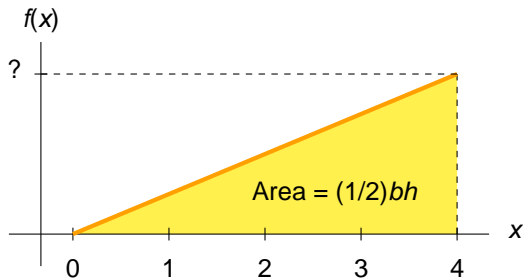
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A Non-Uniform Distribution



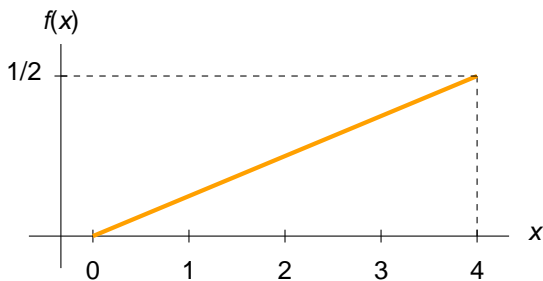
What is the height of this distribution?

A Non-Uniform Distribution



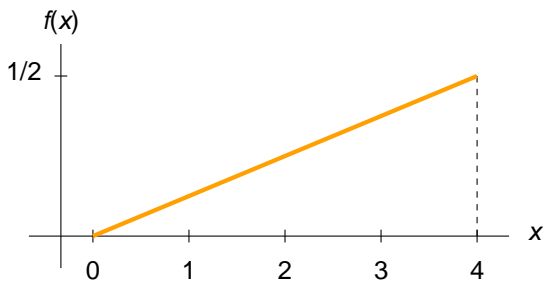
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A Non-Uniform Distribution



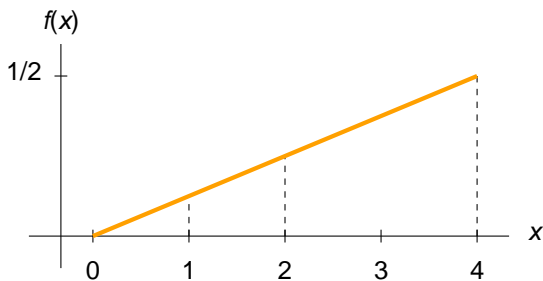
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A Non-Uniform Distribution



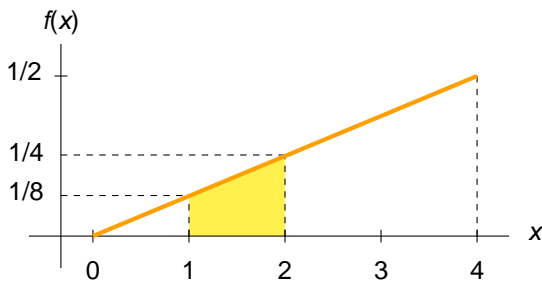
What is the probability that $1 \leq X \leq 2$?

A Non-Uniform Distribution



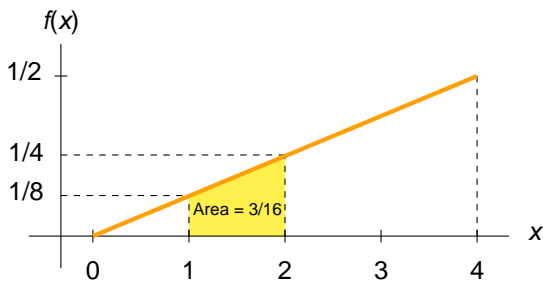
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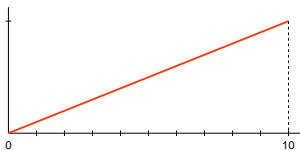
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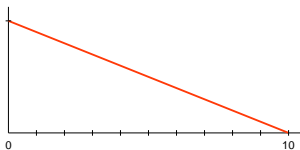
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Bag A vs. Bag B

- Suppose we have two bags, Bag A and Bag B.
- Each bag contains *millions* of vouchers.
- In Bag A, the values of the vouchers have the distribution:



- In Bag B, the values of the vouchers have the distribution:



Bag A vs. Bag B

- We are handed one of the bags.
- The hypotheses are
 - H_0 : It is Bag A.
 - H_1 : It is Bag B.
- We select one voucher at random from one bag.
- Decision Rule: We will reject H_0 if the value of the voucher is less than \$3.
- What is the value of α ?
- What is the value of β ?

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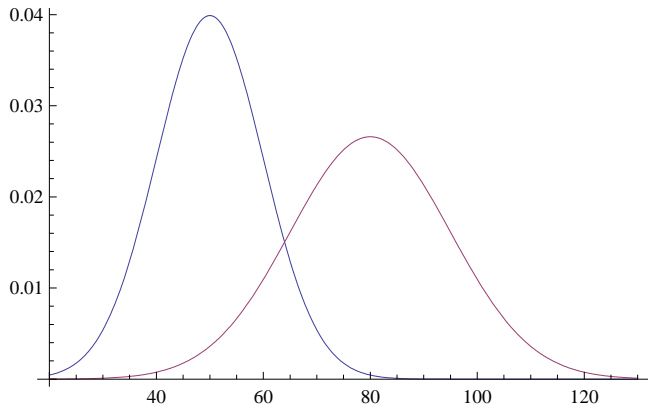
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Drug User vs. Non-drug User

- A subject is either a drug user or a non-drug user.
- To determine which, we measure the level of Substance X in his blood.
- If he is not a drug user, the level of X has the distribution $N(50, 10)$.
- If he is a drug user, the level of X has distribution $N(80, 15)$.

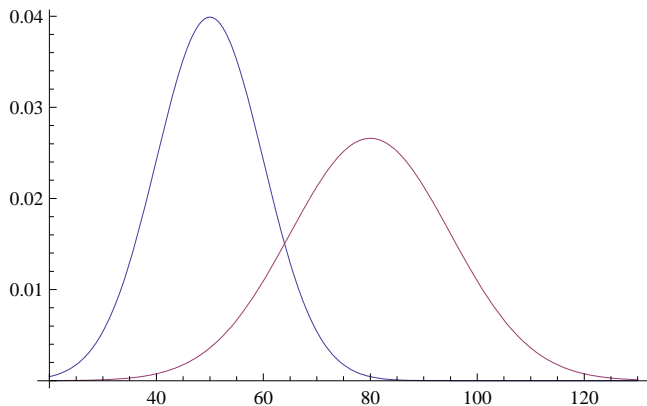
Drug User vs. Non-drug User



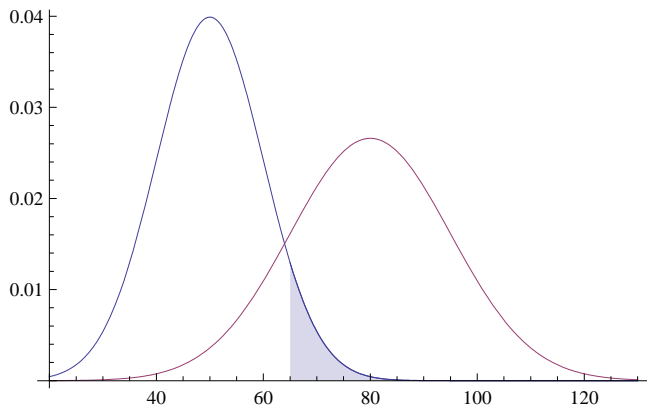
Drug User vs. Non-drug User

- We take a blood sample and measure the amount of X .
- Decision Rule: If the level is more than 65, then we will decide that he is a drug user.
- Describe a Type I error.
- Describe a Type II error.
- What is the value of α ?
- What is the value of β ?

Drug User vs. Non-drug User

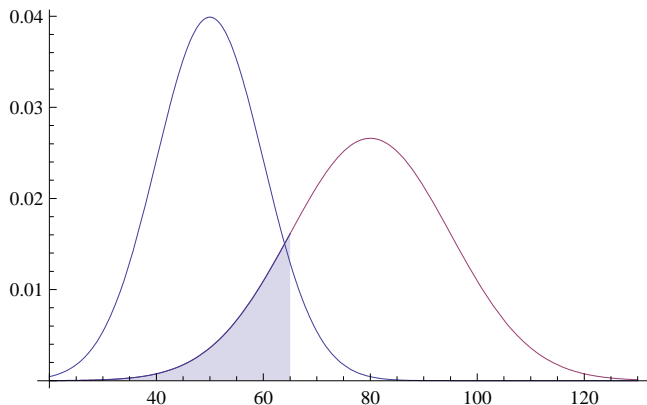


Drug User vs. Non-drug User



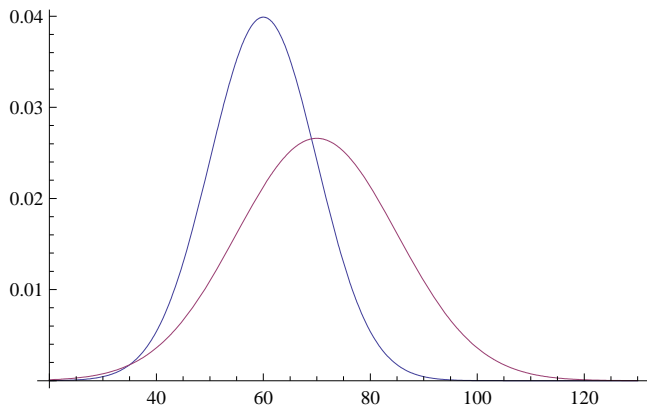
$$\alpha = \text{normalcdf}(65, E99, 50, 10) = 0.0668$$

Drug User vs. Non-drug User



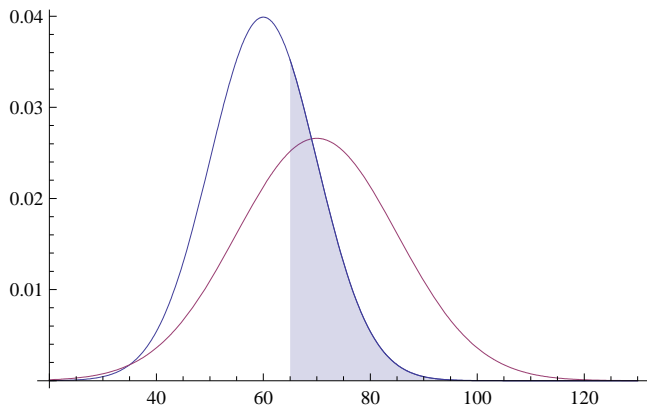
$$\beta = \text{normalcdf}(-E99, 65, 80, 15) = \mathbf{0.1587}$$

Drug User vs. Non-drug User



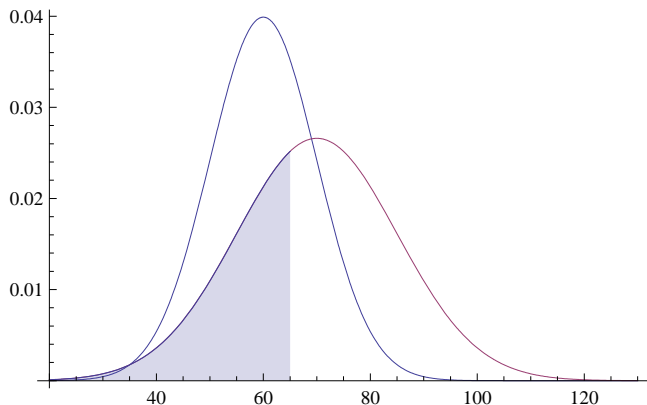
If the means are very close together, then α and β will be large.

Drug User vs. Non-drug User



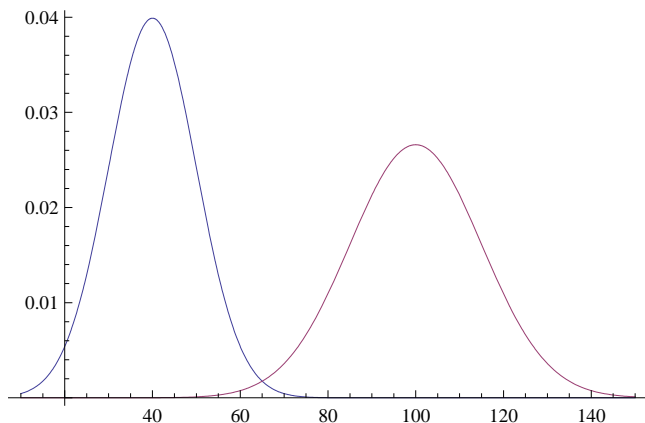
$$\alpha = \text{normalcdf}(65, E99, 60, 10) = 0.3085$$

Drug User vs. Non-drug User



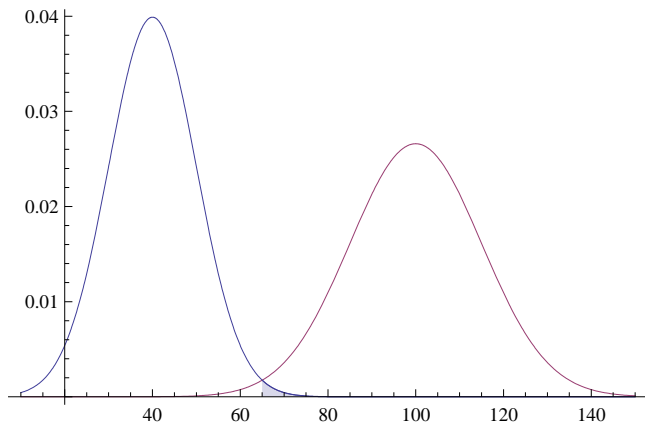
$$\beta = \text{normalcdf}(-E99, 65, 70, 15) = 0.3694$$

Drug User vs. Non-drug User



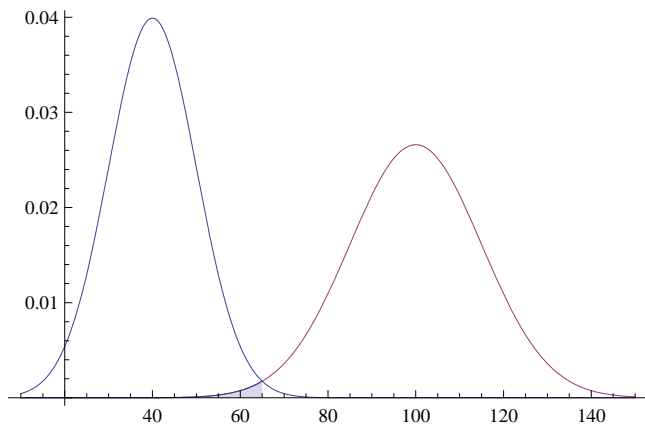
If the means are far apart, then α and β will both be very small.

Drug User vs. Non-drug User



$$\alpha = \text{normalcdf}(65, E99, 40, 10) = 0.0062$$

Drug User vs. Non-drug User



$$\beta = \text{normalcdf}(-E99, 65, 100, 15) = 0.0098$$

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Assignment

Homework

- Read Section 7.5.4, pages 478 - 481.
- Exercises 64, 65, 66a-c, 68, 69, page 481.
- Review Exercises 105, 106, page 489.